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RESONANCE VIBRATIONS IN INTAKE AND EXHAUST PIPES OF IN-LINE ENGINES

Part 3 - The Inlet Process of a Four-Stroke-Cycle Engine

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RESONANCE VIBRATIONS IN INTAKE AND EXHAUST PIPES OF IN-LINE ENGINES*

Part 3 - The Inlet Process of a Four-Stroke-Cycle Engine

By O. Lutz

In the first two parts of this paper (reference 1) resonance phenomena in pipe lines were considered, the assumption being made that the excitation was produced by simple velocity fluctuations (which, in the inlet process, were assumed to be proportional approximately to the piston motion).

The above assumption can represent only a rough approximation which to some extent enables the resonance frequencies to be correctly computed (as in Parts 1 and 2) but which fails to solve the important problem for the inlet process, namely, the effect of the resonance vibrations on the volumetric efficiency. If the latter is affected by the vibrations, the velocity fluctuations at the point of excitation are no longer proportional to the piston motion but are a function of the vibrations. It is evident that any mode of computation which does not take this into account can not be used to explain this phenomenon, although previously this had always been attempted.

Taking the case of resonance vibrations which are excited by a periodically controlled opening, the author had developed a method (reference 2) in which the boundary conditions were made to correspond more nearly to actual conditions than has been done heretofore and by which the change in the flow due to the fluctuations could be computed. In the present paper this method will be extended to the inlet process of a four-stroke-cycle engine. In section 1 the previously developed method will be briefly described. In section 2 application will be made to the

*Über Resonanzschwingungen in den Ansaug- und Auspuffleitungen von Reihenmotoren. 3. Teilbericht: Die Anregungsbedingungen beim Ansaugvorgang von Viertaktmotoren. Luftfahrtforschung, vol. 17, no. 4, April 20, 1940, pp. 123-28.

inlet process and the required boundary conditions obtained. In section 3 the conditions obtained will be applied to the simplest case of a single-cylinder engine and, finally, in section 4 the basic principles will be given for application to multicylinder arrangements.

1. PRINCIPLE OF THE METHOD

The case considered is that sketched in figure 1. At the end of the pipe $x = l$, the flow passes through a periodically controlled opening into a chamber where no back pressure is assumed to be built up. The narrowest flow cross section is denoted by f, the pipe cross section by f_r. With the notation of figure 1 we have

$$\frac{\rho}{2}(v^2 - v_r^2) = p_r - p$$

(where the local acceleration term is neglected). Using the continuity condition:

$$f\,v = f_r\,v_r$$

we have

$$v_r = \frac{f}{\sqrt{f_r^2 - f^2}}\sqrt{\frac{2}{\rho}(p_r - p)} \qquad (1)$$

Defining mean values $\overline{v}_o$, f_o, p_{ro}, and p_o by

$$v_r = \overline{v}_o\left(1 + \frac{\Delta v}{\overline{v}_o}\right), \quad f = f_o\left(1 + \frac{\Delta f}{f_o}\right)$$

$$p_r = p_{ro}\left(1 + \frac{\Delta p_r}{p_{ro}}\right), \quad p = p_o\left(1 + \frac{\Delta p}{p_o}\right)$$

and neglecting products and powers of the relative deviations from the mean values $\frac{\Delta v}{\overline{v}_o}$, $\frac{\Delta f}{f_o}$, etc., we have

$$\overline{v}_o = \frac{f_o}{\sqrt{f_r^2 - f_o^2}}\sqrt{\frac{2}{\rho}(p_{ro} - p_o)} \qquad (2)$$

and

$$\Delta v = \frac{\bar{v}_o \Delta f}{f_o}\left(1 + \frac{\Delta p_r - \Delta p}{2(p_{ro} - p_o)} \ldots\right) + \bar{v}_o\left(\frac{\Delta p_r - \Delta p}{2(p_{ro} - p_o)} \ldots\right) \qquad (3)$$

(For simplification $\sqrt{f_r^2 - f^2}$ has been set equal to $\sqrt{f_r^2 - f_o^2}$ since the jet cross section f will always be small compared to the pipe cross section f_r.)

In the paper cited previously the idea was developed that for the computation of the magnitude and phase of the vibration the linear terms are used while for the computation of the changes in the mean values the terms of higher order were used, terms of second order only being taken into account at first. Thus, setting the mean velocity equal to the sum of the mean value $\bar{v}_o$ from equation (2) for the system without vibrations and the increment Δv_o, which gives the changes by the vibration process:

$$v_o = \bar{v}_o + \Delta v_o \qquad (4)$$

we have

$$\bar{v}_o = \frac{f_o}{\sqrt{f_r^2 - f_o^2}}\sqrt{\frac{2}{\rho}(p_{ro} - p_o)} \qquad (2)$$

$$\Delta v = \frac{\bar{v}_o}{f_o}\Delta f + \frac{v_o}{2(p_{ro} - p_o)}(\Delta p_r - \Delta p) \qquad (5)$$

$$\Delta v_o = \frac{\bar{v}_o}{2f_o(p_{ro} - p_o)}\Delta f(\Delta p_r - \Delta p) \qquad (6)$$

As described in the first part of this paper (reference 1) and in the previous work, the computation is carried out using complex variables, setting

$$\left.\begin{aligned} \Delta v &= \Sigma\, \underline{V}_n{}^{in\omega t} \\ \Delta f &= \frac{f_o}{\bar{v}_o}\, \Sigma\, F_n^* e^{in\omega t} \\ \Delta p_r &= \rho a\, \Sigma\, \underline{P}_{rn}^* e^{in\omega t} \\ \Delta p &= \rho a\, \Sigma\, \underline{P}_n^* e^{in\omega t} \end{aligned}\right\} \qquad (7)$$

(a denotes the velocity of sound; all vibration vectors are converted to the dimensions of a velocity.)

From equation (5) there follows the boundary condition:

$$\underline{V}_n = \underline{F}_n^* + \epsilon\,(\underline{P}_{rn}^* - \underline{P}_n^*) \tag{8}$$

where ϵ is the "opening coefficient" given by

$$\epsilon = \frac{\rho\, a\, \overline{v}_o}{2(p_{ro} - p_o)} = \frac{a\, f_o^2}{\overline{v}_o(f_r^2 - f_o^2)} \tag{9}$$

For the deviation Δv_o from the mean value there is obtained from equation (6)

$$\Delta v_o = \frac{\epsilon}{\overline{v}_o}\,\frac{1}{t_o}\int_0^{t_o = \frac{2\pi}{\omega}} \left(\Sigma\, \underline{F}_n^* e^{in\omega t}\right)\left[\Sigma\,(\underline{P}_{rn}^* - \underline{P}_n^*)\, e^{in\omega t}\right] dt \tag{10}$$

In the previous work it is then proven that

$$\Delta v_o = \frac{\epsilon}{2\overline{v}_o}\; \Sigma\, \underline{F}_n^*(\underline{P}_{rn}^* - \underline{P}_n^*) \tag{10a}$$

where the sum is built up of scalar products. Thus, if the difference vector $\underline{P}_{rn}^* - \underline{P}_n^*$ is out of phase by less than a quarter period with the opening vector $\underline{F}_n^*$, the average quantity of charge delivered is increased by the vibration process. In the previous work examples for this case of excitation were computed and agreement of the test results with the computation proven.

2. APPLICATION OF THE METHOD TO THE INLET PROCESS

The inlet process in the case of a four-stroke-cycle engine can be divided up into two processes, the first of which was already considered in section 1, namely, the flow process through the controlled opening and the charging process inside the cylinder.

For the first process the boundary condition is already known:

$$\underline{V}_r = \underline{F}^* + \epsilon\,(\underline{P}_r^* - \underline{P}^*) \tag{8a}$$

(written down for the fundamental); the second process requires further consideration.

For an adiabatic charging process, we have, with the notation of figure 2:

$$\frac{dp}{p} = \kappa \frac{dG}{G} - \kappa \frac{dJ}{J} \tag{11}$$

where G is the variable weight of the gas, J the instantaneous cylinder volume. The rate of change with time is

$$\frac{dG}{dt} = \gamma \frac{dJ}{dt} + \frac{\gamma}{\kappa p} J \frac{dp}{dt}$$

Converting the cylinder volume into pipe lengths by

$$s = \frac{J}{f_r}$$

there is obtained with $\frac{dG}{dt} = \gamma f_r v_r$:

$$v_r = \frac{ds}{dt} + \frac{s}{\kappa p} \frac{dp}{dt} \tag{12}$$

If no pressure fluctuations were set up in the cylinder, the boundary condition would be

$$v_r = \frac{ds}{dt} \tag{12a}$$

As in section 1, the mean values and the deviations from those mean values

$$\frac{ds}{dt} = c_o\left(1 + \frac{\Delta c}{c_o}\right) \quad s = s_o\left(1 + \frac{\Delta s}{s_o}\right)$$

are again substituted in equation (12).

$$v_o\left(1 + \frac{\Delta v}{v_o}\right) = c_o\left(1 + \frac{\Delta c}{c_o}\right) + \frac{s_o}{\kappa p_o} \frac{1 + \frac{\Delta s}{s_o}}{1 + \frac{\Delta p}{p_o}} \frac{d\Delta p}{dt} \tag{12b}$$

and the vibration magnitudes as in equation (7) expressed in complex form:

$$\left.\begin{aligned}\Delta c &= \Sigma\, \underline{C}_n e^{in\omega t}\\ \Delta s &= \Sigma\, \underline{S}_n e^{in\omega t}\end{aligned}\right\} \tag{13}$$

The linear terms then lead to the boundary condition (written for the nth harmonic):

$$\underline{V}_{rn} = \underline{C}_n + i\,\frac{n\omega s_o}{a}\,\underline{P}_n^* \tag{14}$$

The terms of second order give for the mean values:

$$v_o = c_o + \frac{1}{\rho a^2}\,\Delta s\,\frac{d(\Delta p)}{dt} - \frac{s_o}{\rho a^2 p_o}\,\Delta p\,\frac{d\Delta p}{dt} \tag{15}$$

In the same manner as for equations (10) and (10a), setting $\Delta v_o = v_o - c_o$, there is finally obtained

$$\Delta v_o = \frac{\omega}{2a}\,\Sigma\,\underline{S}_n\; i\; n\; \underline{P}_n^* \tag{15a}$$

(The last term in equation (15) gives no contribution since the first derivative of the function itself leads by a quarter period, so that the scalar product of the two vanishes.)

Between the harmonics $\underline{S}_n$, which give the cylinder volume at any instant, and the harmonics $\underline{C}_n$, which give the instantaneous piston velocity, there is a definite relation. In figure 5 the periodic relation is drawn for a four-stroke-cycle engine (effect of finite connecting rod neglected). The fundamental frequency of the inlet process (that is, half the engine frequency) is denoted by ω.

We have

$$c = \frac{ds}{dt}\,, \quad \text{thus} \quad c_1 = 2\omega\, s_1 \tag{16}$$

Expansion into a harmonic series gives

$$\left.\begin{aligned} s &\doteq s_o + \frac{\sqrt{2}}{3\pi} s_1 \sin\omega t + \frac{s_1}{4} \sin 2\omega t + \frac{3\sqrt{2}}{5\pi} s_1 \sin 3\omega t \\ &\quad + \frac{2}{3\pi} s_1 \sin 4\omega t \ldots\ldots \\ &\text{and} \\ c &= \frac{1}{2\pi} c_1 + \frac{2\sqrt{2}}{3\pi} c_1 \cos\omega t + \frac{c_1}{4} \cos 2\omega t \\ &\quad + \frac{2\sqrt{2}}{5\pi} c_1 \cos 3\omega t + \frac{1}{3\pi} c_1 \cos 4\omega t \ldots . \end{aligned}\right\} \quad (17)$$

from which, using equation (16), there is obtained

$$\frac{d\underline{S}_n}{dt} = \frac{n^2}{4} \underline{C}_n \quad (18)$$

or

$$i\omega\underline{S}_n = \frac{n}{4} \underline{C}_n \quad (18a)$$

Equation (15a) then goes over into

$$\Delta v_o = -\frac{n^2}{8a} \Sigma \underline{C}_n \underline{P}_n^* \quad (15b)$$

The boundary condition at the cylinder end of the pipe is thus given by the vector equations

$$\underline{V}_{rn} = \underline{E}_n^* + \epsilon(\underline{P}_{rn}^* - \underline{P}_n^*) \quad (8)$$

and

$$\underline{V}_{rn} = \underline{C}_n + i\frac{n\omega s_o}{a} \underline{P}_n^* \quad (14)$$

while the change in the volumetric efficiency due to the vibration process is given by

$$\Delta v_o = -\frac{n^2}{8a} \Sigma \underline{C}_n \underline{P}_n^* \quad (15b)$$

It should be pointed out once more that n gives the order of the harmonic and is not a rotational speed.

Between the "opening coefficient" ϵ in equation (8) and the frequency ω for the inlet process of a four-stroke-cycle engine there is a hyperbolic relation. For, from equation (9),

$$\epsilon = \frac{a\ f_o^2}{\bar{v}_o(f_r^2 - f_o^2)} \tag{9}$$

The mean pipe velocity $\bar{v}_o$ (without taking account of the vibration effect) is proportional, however, to the engine speed and thus to the fundamental frequency ω. Thus, from equation (17),

$$\bar{v}_o = \frac{c_1}{2\pi} \tag{17a}$$

whence, using equation (16),

$$\bar{v}_o = \frac{\omega s_1}{\pi}$$

According to figure 3, the stroke amplitude s and the mean value s_o are connected by the relation

$$\frac{s_o + s_1}{s_o - s_1} = \epsilon_v$$

(ϵ_v is the compression ratio), so that, finally,

$$\bar{v}_o = \frac{\epsilon_v - 1}{\epsilon_v + 1}\,\frac{\omega s_o}{\pi} \tag{17b}$$

and for the opening coefficient there is obtained

$$\epsilon = \frac{\beta}{\dfrac{\omega s_o}{a}} \tag{19}$$

with

$$\beta = \frac{\pi(\epsilon_v + 1)f_o^2}{(\epsilon_v - 1)(f_r^2 - f_o^2)} \tag{20}$$

3. EXAMPLE: SINGLE-CYLINDER ENGINE

Let the pipe connecting to the single-cylinder engine be of length l (fig. 4). As proven in the first paper, for an open pipe the following relations exist between the vibration magnitudes

$$\left.\begin{aligned} \underline{P}_r^* &= -i \tan \frac{\omega l}{a} \underline{V}_r \\ \text{and} \qquad & \\ \underline{V}_o &= \frac{1}{\cos \frac{\omega l}{a}} \underline{V}_r \end{aligned}\right\} \tag{21}$$

With boundary conditions (8) and (14), written down for the fundamental, and taking account of (19), there is obtained after elimination of the magnitudes $\underline{P}_r^*$, $\underline{P}^*$, and $\underline{V}_r$:

$$\underline{V}_o \left(\cos \frac{\omega l}{a} - \frac{\omega s_o}{a} \sin \frac{\omega l}{a} + \frac{1}{\beta} \left(\frac{\omega s_o}{a} \right)^2 \cos \frac{\omega l}{a} \right) = \underline{C} + \frac{1}{\beta} \left(\frac{\omega s_o}{a} \right)^2 \underline{F}^* \tag{22}$$

for $s_o = 0$, the equation assumes the known form

$$\underline{V}_o \cos \frac{\omega l}{a} = \underline{C} \tag{22a}$$

Equation (22) describes the vibration processes in a general form. If the dimensions of the system are known, the entire process can be followed with the aid of this equation. (In a later report an example will be computed.)

The conclusions in the present paper will be obtained with the aid of a simplified example.

It will be assumed that

$$s_o = \frac{l}{2} \quad \text{and} \quad \beta = 1 \tag{23}$$

and, further, that the controlled opening follows the variation in the piston speed in figure 3. Since all the vibration magnitudes are reduced to the dimensions of a velocity, we have in this case simply

$$\underline{F}^* = \underline{C} \tag{23a}$$

Finally, let the nondimensional expression $\frac{\omega l}{a} = \varphi$. The boundary conditions (8) and (14) then become (for the fundamental)

$$\underline{V}_r = \underline{C} + \frac{2}{\varphi}\left(\underline{P}_r^* - \underline{P}^*\right) \tag{8a}$$

and

$$\underline{V}_r = \underline{C} + i\,\frac{\varphi}{2}\,\underline{P}^* \tag{14a}$$

and we have in place of (22)

$$\underline{V}_o\left(\cos\varphi - \frac{\varphi}{2}\sin\varphi + i\,\frac{\varphi^2}{4}\cos\beta\right) = \underline{C}\left(1 + i\,\frac{\varphi^2}{4}\right) \tag{22b}$$

from which, with the aid of (21), there is obtained

$$\underline{V}_r = \frac{1 - \frac{\varphi}{2}\tan\varphi + \frac{\varphi^4}{16}}{\left(1 - \frac{\varphi}{2}\tan\right)^2 + \frac{\varphi^4}{16}}\,\underline{C} - i\,\frac{\frac{\varphi^3}{8}\tan\varphi}{\left(1 - \frac{\varphi}{2}\tan\varphi\right)^2 + \frac{\varphi^4}{16}}\,\underline{C} \tag{24}$$

From (14a) there is further obtained

$$\underline{P}^* = \frac{-\frac{\varphi^2}{4}\tan\varphi}{\left(1 - \frac{\varphi}{2}\tan\varphi\right)^2 + \frac{\varphi^4}{16}}\,\underline{C} - i\,\frac{\tan\varphi - \frac{\varphi}{2}\tan^2\varphi}{\left(1 - \frac{\varphi}{2}\tan\varphi\right)^2 + \frac{\varphi^4}{16}}\,\underline{C} \tag{25}$$

From the above equation (25) and equation (15b) the effect of the fundamental on the change in the volumetric efficiency can be obtained.

We have

$$\Delta v_o = -\frac{\underline{C}\underline{P}^*}{8a}$$

which, with the aid of equation (25) becomes

$$\Delta v_o = \frac{\frac{\varphi^2}{32a}\tan\varphi}{\left(1 - \frac{\varphi}{2}\tan\varphi\right)^2 + \frac{\varphi^4}{16}}\,\underline{C}_1{}^2$$

where C_1 denotes the absolute value of the amplitude of the fundamental vibration.

From equation (17)

$$C_1 = \frac{2\sqrt{2}}{3\pi} c_1$$

and from (17a)

$$C_1 = \frac{4\sqrt{2}}{3} \bar{v}_0$$

and using the relations that led to formula (19)

$$\frac{\Delta v}{\bar{v}_0} = \frac{4}{9\pi} \frac{\epsilon_v - 1}{\epsilon_v + 1} \frac{\frac{\varphi^3}{8} \tan \varphi}{\left(1 - \frac{\varphi}{2} \tan \varphi\right)^2 + \frac{\varphi^4}{16}} \tag{26}$$

The results can best be represented graphically. In figure 5 the relation between the piston velocity fluctuation and the velocity fluctuation V_r according to (24) is given by a polar diagram. The deviation from the usual assumption that V_r is equal to C is, for the example chosen, often considerable. The phase shifts can amount to more than a quarter period and considerable differences arise also in the amplitudes which differences may be particularly large in the frequency range before the first resonance $(0 < \varphi < 90^\circ)$.

Figure 6 shows the variation in the velocity fluctuation $\underline{V}_0$ at the pipe entrance (according to (21)). It is interesting that no marked resonances occur although no damping has been assumed. Whereas, with the usual assumption $\underline{V}_r = \underline{C}$, the velocity fluctuation $\underline{V}_0'$ is in phase with $\underline{C}$ (shown dotted) and becomes infinitely great at the resonance positions (90°, 270°) the phase position of $\underline{V}_0$ changes continuously but no infinitely large amplitudes any longer occur. The oval-shaped loci described by the tip of the vector $\underline{V}_0$ take a surprising course. The first oval, which is for $0 < \varphi < 180^\circ$, is large; the second $(180^\circ < \varphi < 360^\circ)$ is small. From then on, however, the ovals become increasingly large. Examining the relations more closely, we find that the valve motion is mainly responsible for this increase in the vibration values. It may therefore be concluded that each change in the valve motion must have an important effect on the process. For large frequencies, that is, for large values of φ or $\omega l/a$, the linear and the quadratic terms in equation (22) are predominant, so that equation (22) may be replaced by

$$\underline{V}_o\left(-\frac{\omega s_o}{a}\sin\frac{\omega l}{a}+\frac{i}{\beta}\left(\frac{\omega s_o}{\alpha}\right)^2\cos\frac{\omega l}{a}\right)=\frac{i}{\beta}\left(\frac{\omega s_o}{a}\right)^2\underline{F}^*$$

or

$$\underline{V}_o\left(\cos\frac{\omega l}{a}+i\frac{\beta}{\frac{\omega s_o}{a}}\sin\frac{\omega l}{a}\right)=\underline{F}^* \tag{22c}$$

Without giving a detailed proof (reference 2, p. 37), an equation of the form (22c) describes the process at the entrance of a throttled pipe line (fig. 7) which at the other end is excited by a fluctuation in the size of the opening $\underline{F}^*$. The factor $\dfrac{\beta}{\frac{\omega s_o}{a}}$ corresponds to the "throttling coefficient" δ_1, which, in the reference cited, was derived as

$$\delta_1=\frac{f_r^2}{f_1^2}\frac{v_o}{a}=\frac{v_d^2}{av_o}$$

At the higher frequencies therefore the system may be replaced by that shown in figure 7, where the throttling, it may be remarked, is to be chosen smaller with increasing frequencies.

In figure 8, finally, the variation of the volumetric efficiency $\dfrac{\Delta v_o}{v_o}$ is shown as a function of $\varphi=\dfrac{\omega l}{a}$, if a compression ratio of $\epsilon_v = 6$ is assumed. It is found that the vibration may give rise to improvement as well as to impairment of the volumentric efficiency. The amplitudes $\underline{V}_o$ of the velocity fluctuations at the pipe entrance (from fig. 6) are also drawn. The first resonance position occurs ahead of the value (90°) valid for the assumption $\underline{V}_r = \underline{C}$, the difference amounting to 33 percent. The subsequent resonance positions, however, differ only slightly (270°, 450°, 630°...). The figure also shows clearly the increase in the maximum value at the higher resonance positions (lesser throttling). The gains in volumetric efficiency always occur ahead of the vibration resonance positions, become equal to zero at the resonance positions, and become negative after the resonance positions. In the example chosen, the best value of the gain is about 30 percent and at the higher resonance positions no more than 6 or 7 percent. If the fact is taken into account that the vibration process is assumed to be without losses (frictional and damping), no important effects should be expected, at least at the high resonances.

A later report will give the results of an investigation made with the aid of examples to determine how strong is the effect of certain factors, particularly a change in the timing of the opening cross section.

For the present the following conclusions may be drawn:

1. The inlet process in the case of a four-stroke-cycle engine leads to vibrations which may considerably differ from those that are excited by a pure velocity fluctuation. With open pipes no infinitely large vibrations occur.

2. The frequencies of the large vibrations (resonance frequencies) are lower at the first resonance position than in the case of velocity excitation and the intake pipe thus appears as elongated. The higher resonances, however, are in good agreement.

3. At higher frequencies the system operates like a pipe throttled at the entrance, which at its other end is excited with a velocity fluctuation equal to the opening fluctuation, and it is necessary that the throttling be reduced with increasing frequency.

4. As a result of the vibration the volumetric efficiency can be considerably affected.

5. A gain in the volumetric efficiency always takes place ahead of the resonance positions (in the sense of increasing values of $\omega l/a$). It is largest before the first resonance position (30 percent for the example chosen). Beyond the resonance positions the volumetric efficiency is impaired.

4. MULTICYLINDER ENGINES

The treatment of multicylinder engines presents no fundamental difficulties. The phenomena soon become so complicated, however, that the general treatment promises few results of fundamental importance. The individual treatment of a given system is not difficult. The relations for a two-cylinder system will be briefly developed.

The intake piping of a two-cylinder engine is sketched in figure 9. It is known from the previous paper that the process can be simply followed if the pipes are considered as made up of simple "open" pipe parts, as shown in figure 10. Pipe 1 (length l) has the required velocity fluctuation $\underline{V}_o$ at its entrance; the pressure fluctuation $\underline{P}_o{}^*$ can be set equal to zero (open pipe). At the end of this pipe the fluctuations are given by the following known relations

$$\underline{P}^{*\,\prime}_{r3} = -\ i \sin \frac{\omega l}{a} \underline{V}_o$$

$$\underline{V}'_{II} = \cos \frac{\omega l}{a} \underline{V}_o$$

Pipe 2 (length l_2) is also "open" at the entrance since at this point the pressure fluctuation to be superimposed on that of pipe 1 is $\underline{P}^*_{r1} = 0$; the exciting velocity fluctuation (to be determined) is $\underline{V}_I$. Corresponding to these initial conditions, the fluctuations at the other end are given by

$$\underline{P}^{*\,\prime\prime}_{r3} = +\ i \sin \frac{\omega l_2}{a} \underline{V}_I$$

$$\underline{V}''_{II} = -\ \cos \frac{\omega l_2}{a} \underline{V}_I$$

The fluctuations of the entire system at the other end are therefore

$$\underline{P}^*_{r3} = -i \sin \frac{\omega l}{a} \underline{V}_o + i \sin \frac{\omega l_2}{a} \underline{V}_I \qquad (27a)$$

$$\underline{V}_{II} = \cos \frac{\omega l}{a} \underline{V}_o - \cos \frac{\omega l_2}{a} \underline{V}_I \qquad (27b)$$

It is now necessary to find the relation between the fluctuation values of the velocity, the pressure, and the intake system (piston velocity and control opening).

Boundary conditions (8) and (14) using the opening coefficient ϵ given by equation (19), may be written (for the fundamental):

$$\underline{V}_r = \underline{F}^* + \frac{\beta}{\frac{\omega s_o}{a}} \left(\underline{P}_r{}^* - \underline{P}^*\right) \tag{8b}$$

and

$$\underline{V}_r = \underline{C} + i\,\frac{\omega s_o}{a}\,\underline{P}^* \tag{14b}$$

Eliminating $\underline{P}^*$, the relation between $\underline{V}_r$ and $\underline{P}_r{}^*$:

$$\underline{V}_r = \frac{\underline{C} + i\,\frac{\omega^2 s_o{}^2}{\beta a^2}\,\underline{F}^*}{1 + i\,\frac{\omega^2 s_o{}^2}{\beta a^2}} + \frac{i\,\frac{\omega s_o}{a}}{1 + i\,\frac{\omega^2 s_o{}^2}{\beta a^2}}\,\underline{P}_r{}^* \tag{28}$$

and setting

$$\underline{a}\,\underline{C} = \frac{\underline{C} + i\,\frac{\omega^2 s_o{}^2}{\beta a^2}\,\underline{F}^*}{1 + i\,\frac{\omega^2 s_o{}^2}{\beta a^2}} \tag{29}$$

and

$$\underline{b} = \frac{i\,\frac{\omega s_o}{a}}{1 + i\frac{\omega^2 s_o{}^2}{\beta a^2}} \tag{29a}$$

the relation may be put in the general form

$$\underline{V}_r = \underline{a}\,\underline{C} + \underline{b}\,\underline{P}_r{}^* \tag{28a}$$

If equal cylinders are assumed, with the arrangement of figure 9, the values a and b for both points of excitation are equal, and the following relations are obtained

$$\underline{V}_I = \underline{a}\ \underline{C}_I + \underline{b}\ \underline{P}^*_{r1} \tag{27c}$$

and

$$\underline{V}_{II} = \underline{a}\ \underline{C}_{II} + \underline{b}\ \underline{P}^*_{r3} \tag{27d}$$

The pressure fluctuation which occurs in the open pipe portion 1 at the distance l_1 from the pipe entrance is given by

$$\underline{P}^*_{r1} = -\, i \sin \frac{\omega l_1}{a}\ \underline{V}_0 \tag{27e}$$

From the five relations ((27a) to (27e)) thus set up, the five unknowns $\underline{V}_0$, $\underline{V}_I$, $\underline{V}_{II}$, $\underline{P}^*_{r1}$, and $\underline{P}_r{}^*$ may be determined.

In the same manner every system can be investigated. General conclusions are difficult to draw because the coefficients a and b are directed magnitudes (vectors) which are not constant but vary with the frequency.

5. SUMMARY

Using a previously developed method, the boundary conditions for the resonance vibrations during the inlet process of four-stroke-cycle engines are set up. The results deviate considerably from those obtained under the assumption that the velocity fluctuation is proportional to the cylinder piston motion. The deviation is less at the position of resonance frequencies. By the method developed, the effect of the resonance vibrations on the volumetric efficiency can be demonstrated.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

REFERENCES

1. Lutz, O.: Über Resonanzschwingungen in den Ansaug- und Auspuffleitungen von Reihenmotoren.
   Part 1.- Die Vorgänge in unverzweigten, an mehreren Stellen angeregten Leitungen. Luftfahrtforschung, vol. 16, no. 3, March 20, 1939, pp. 139-47.
   Part 2.- Die Vorgänge in verzweigten Leitungen. vol. 16, no. 7, July 20, 1939, pp. 384-90.

2. Lutz, O.: Resonanzschwingungen in den Rohrleitungen von Kolbenmaschinen, Heft 3 der Berichte aus dem Laboratorium für Verbrennungskraftmaschinen der TH Stuttgart, Stuttgart 1934.

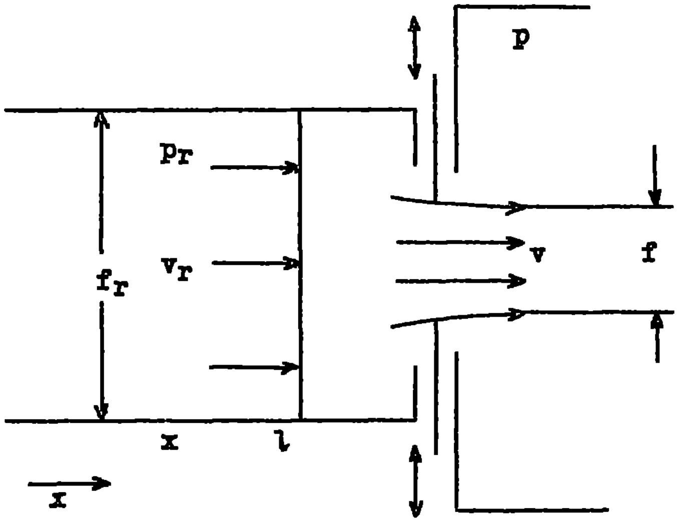
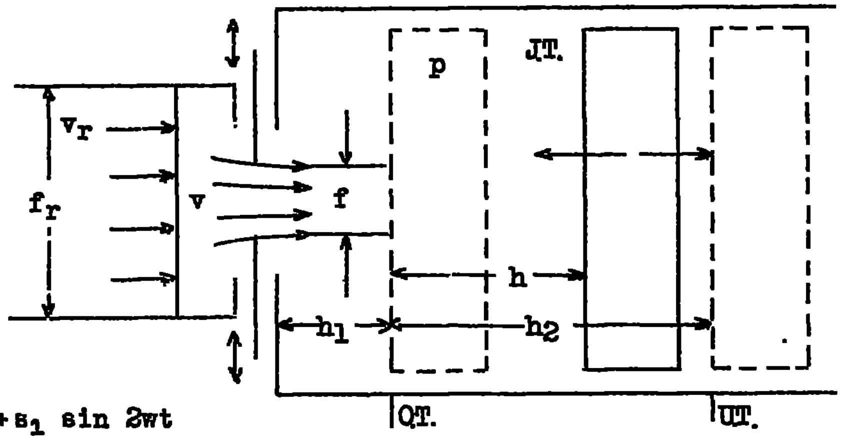


Figure 1.- Sketch for setting up the boundary condition (8).

Figure 2.- Sketch for setting up the boundary condition for the inlet process.



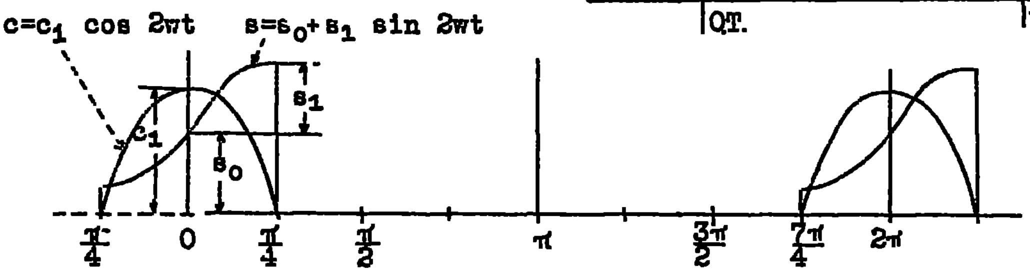


Figure 3.- Piston velocity and cylinder volume during the inlet process.

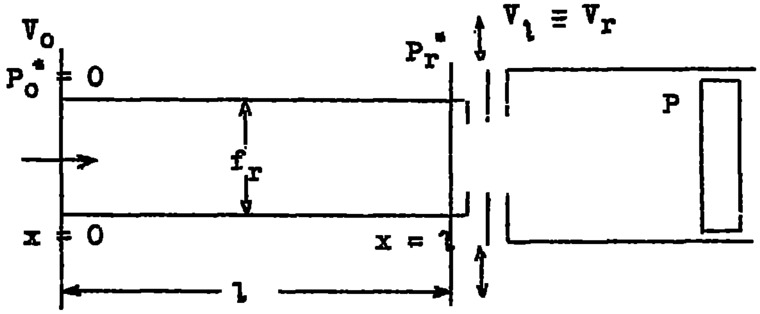


Figure 4.- Intake pipe to a single-cylinder engine.

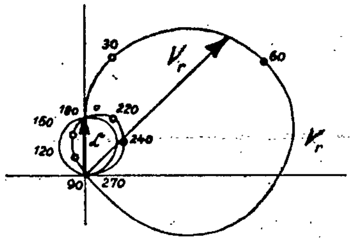


Figure 5.- Relation between the piston velocity fluctuation $\dot{\xi}$ and the velocity fluctuation V_r (assumption made: $V_r = C$).

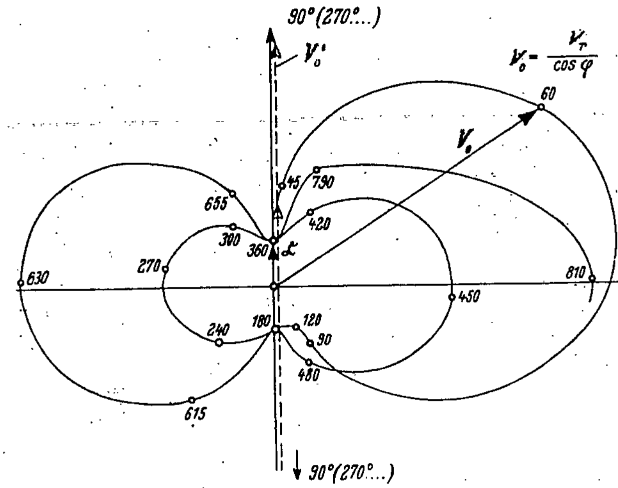
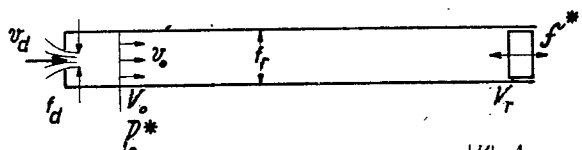


Figure 6.- Velocity fluctuation V_0 at the pipe entrance (V_0' holds for the usual assumption $V_r = C$).

Figure 7.- Equivalent system.



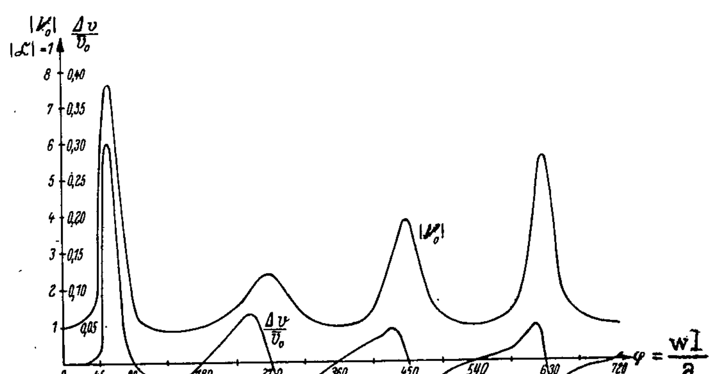


Figure 8.- Change in the volumetric efficiency due to the vibration process.

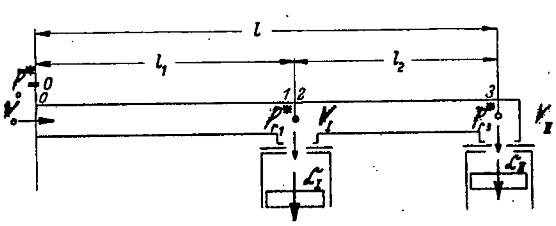


Figure 9.- Schematic two-cylinder arrangement.

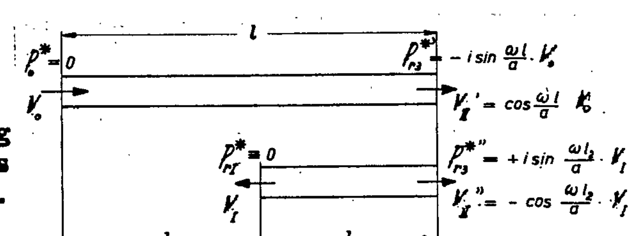


Figure 10.- Method of considering the pipe in Fig. 9 as divided into two open pipe parts.